Int J Theor Phys (2008) 47: 1552–1558
DOI 10.1007/s10773-007-9596-7

Classical Communication Help and Probabilistic Teleportation with One-Dimensional Non-maximally Entangled Cluster States

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Received: 12 May 2007 / Accepted: 17 October 2007 / Published online: 20 November 2007

Abstract We present an explicit protocol for probabilistic teleport an arbitrary and unknown two-qubit entangled state via a one-dimensional four-particle non-maximally entangled cluster state. By construction, our four-partite state is not reducible to a pair of Bell states. We show that teleportation can be successfully realized with a certain probability. This protocol indicate that the four-qubit state is a likely candidate for the genuine four-particle analogue to a Bell state.

Keywords Genuine multipartite entangled state · Probabilistic teleportation · Two-qubit entangled state

No-cloning theorem forbids a perfect copy of an arbitrary unknown quantum state. How to interchange different resources has ever been a question in quantum computation and quantum information. Quantum entanglement and classical communication are two elementary resources in quantum information field. Quantum teleportation [2–14], the disembodied transport of quantum states between subsystems through a classical communication channel requiring a shared resource of entanglement, is one of the most profound results of quantum information theory [1]. Quantum teleportation process, originally proposed by Bennett et al. [2], can transmit an unknown quantum state from a sender to a spatially distant receiver via a quantum channel with the help of some classical information. Their work showed in essence the interchangeability of different resources in quantum mechanics. Later, quantum teleportation has received much attention [2–14] both theoretically and experimentally in recent years due to its important applications in quantum communications. For example, in 1998, Karlsson and Bourennane [3] generalized Bennett et al.'s teleportation idea by using a 3-qubit Greenberger–Horne–Zeilinger (GHZ) state $|000\rangle + |111\rangle$ instead of an EPR pair.

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Dai et al. [13, 14] have proposed two schemes to teleport an arbitrary two-particle state by two non-maximally three-particle entangled W state, as well as the combination of a non-maximally three-particle entangled GHZ state and a non-maximally three-particle entangled W state, respectively, and the teleportation of an arbitrary two-qubit state had been studied by Lee et al. [6] and recently by Rigolin [7].

In this paper, we give an explicit protocol for faithfully teleporting arbitrary two-qubit states employing one-dimensional four-qubit non-maximally entangled cluster states. Since our work is motivated in part by protocol [2], we briefly describe it below before presenting our protocol. This is followed by a detailed analysis on the entanglement properties of $|\chi\rangle_{A_3A_4B_1B_2}$, where we compare and contrast with those of the four party Greenberger–Horne–Zeilinger (GHZ) [15] and W [16] states.

In [2], if Alice and Bob share an Einstein–Podolsky–Rosen (EPR) pairs as follows

$$|\Psi^0_{Bell}\rangle_{A_2B} = \frac{1}{\sqrt{2}}(|00\rangle_{A_2B} + |11\rangle_{A_2B}). \tag{1}$$

Alice can teleport an intact quantum state (2)

$$|\psi\rangle_{A_1} = a|0\rangle_{A_1} + b|1\rangle_{A_1}, \tag{2}$$

to Bob. The initial complete state of the three particles, A_1, A_2, and B, is a pure product state,

$$|\psi\rangle_{A_1A_1}\langle\psi| \otimes |\Psi^0_{Bell}\rangle_{A_2BA_2B}\langle\Psi^0_{Bell}|, \tag{3}$$

involving neither classical correlation nor quantum entanglement between particle A_1 and the maximally entangled pair A_2B. Alice cleanly divides the full information encoded in $|\psi\rangle_{A_1}$ into two parts, transmitting first the purely nonclassical part via the quantum channel $|\Psi^0_{Bell}\rangle_{A_2B}$. Alice first carries out a von Neumann measurement in the Bell basis:

$$|\Psi^i_{Bell}\rangle_{A_1A_2} = (\sigma^i_{A_1} \otimes \sigma^0_{A_2})|\Psi^0_{Bell}\rangle_{A_1A_2}, \tag{4}$$

on the joint system consisting of particles A_1 and A_2. Here, $\sigma^0 = I_2$ is the two-dimensional identity and $\sigma^1 = \sigma_x$, $\sigma^2 = i\sigma_y$ and $\sigma^3 = \sigma_z$. The density operator of Bob's qubit ρ^i_B conditioned on Alice's Bell measurement outcome i is

$$\begin{aligned} &\frac{1}{P_i}\mathrm{tr}_{A_1A_2}[(|\psi\rangle_{A_1A_1}\langle\psi| \otimes |\Psi^0_{Bell}\rangle_{A_2BA_2B}\langle\Psi^0_{Bell}|)(|\Psi^i_{Bell}\rangle_{A_1A_2A_1A_2}\langle\Psi^i_{Bell}|) \otimes I_B)] \\ &\quad = \frac{1}{P_i}\ {}_{A_1A_2}\langle\Psi^0_{Bell}|(\sigma^i_{A_i}|\psi\rangle_{A_i} \otimes |\Psi^0_{Bell}\rangle_{A_2B})({}_{A_i}\langle\psi|\sigma^i_{A_i} \otimes_{A_2B} \langle\Psi^0_{Bell}|)|\Psi^0_{Bell}\rangle_{A_1A_2} \\ &\quad = \frac{1}{4P_i}\sigma^i_B|\psi\rangle_{BB}\langle\psi|\sigma^i_B, \end{aligned} \tag{5}$$

where $P_i = \mathrm{tr}[(|\psi\rangle_{A_1A_1}\langle\psi| \otimes |\Psi^0_{Bell}\rangle_{A_2BA_2B}\langle\Psi^0_{Bell}|)(|\Psi^i_{Bell}\rangle_{A_1A_2A_1A_2}\langle\Psi^i_{Bell}| \otimes I_B)] = 1/4$. It follows that, regardless of the unknown state $|\psi\rangle_{A_1}$, the four measurement outcomes are equally likely. Alice gains no information about the state $|\psi\rangle_{A_1}$ from her measurement. Alice sending two bits of classical information to Bob via a classical channel, after which Bob applies the required Pauli rotation to transform the state of his particle B into an accurate replica of the original state of Alice's particle A_1.

Table 1 Corresponding relations between ij and κ. ($\pm^1$, $\pm^2$, $\pm^3$, $\mp^1$ and $\mp^2$ correspond to the superscripts for the states composed of particles $A_1A_2A_3A_4$)

ω_{ij}^{κ}	$\kappa=1$	$\kappa=2$	$\kappa=3$	$\kappa=4$
$ij=00$	$a_{00}a$	$\pm^2 a_{01}a$	$\pm^1 a_{10}a$	$\pm^1\pm^2 a_{11}a$
$ij=01$	$\pm^2 a_{01}b$	$a_{00}b$	$\pm^1\pm^2 a_{11}b$	$\pm^1 a_{10}b$
$ij=10$	$\pm^1 a_{10}c$	$\pm^1\pm^2 a_{11}c$	$a_{00}c$	$\pm^2 a_{01}c$
$ij=11$	$\pm^1\mp^2 a_{11}d$	$\mp^1 a_{10}d$	$\mp^2 a_{01}d$	$-a_{00}d$

Let us turn to depict our protocol. We propose a protocol for probabilistic teleport an arbitrary and unknown two-qubit entangled state via a one-dimensional four-qubit non-maximally entangled cluster state. To avoid our four qubit entangled channel from being reducible to a tensor product of two Bell states, and to ensure the success of faithfully teleporting any arbitrary two-qubit state, without loss of generality, Alice and Bob share a priori two particles A_3A_4 and B_1B_2 in the non-maximally entangled cluster state

$$|\chi\rangle_{A_3A_4B_1B_2} = (a|0000\rangle + b|0011\rangle + c|1100\rangle - d|1111\rangle)_{A_3A_4B_1B_2}, \tag{6}$$

where the coefficients a, b, c, and d satisfy $|a|^2+|b|^2+|c|^2+|d|^2=1$, $|d|$ is smaller than the absolute value of the other coefficients.

The arbitrary and unknown two-particle state that will be teleported can be expressed as

$$|\Psi\rangle_{A_1A_2} = \sum_{i=0,j=0}^{1} a_{ij}|ij\rangle_{A_1A_2}, \tag{7}$$

with $a_{ij}=C^1$ and $\sum_{i=0,j=0}^{1}|a_{ij}|^2=1$. In order to realize teleportation, we may construct the following basis of 16 orthonormal states (similar to (4)):

$$\begin{aligned} \left|\bar{\prod}^{00}\right\rangle_{A_1A_2A_3A_4} &= |\Psi^0_{Bell}\rangle_{A_1A_2}\otimes|\Psi^0_{Bell}\rangle_{A_3A_4}, \\ \left|\bar{\prod}^{ij}\right\rangle_{A_1A_2A_3A_4} &= [(\sigma^i_{A_1}\otimes\sigma^j_{A_2})\otimes I_{A_3A_4}]\left|\prod{}^{00}\right\rangle_{A_1A_2A_3A_4}. \end{aligned} \tag{8}$$

If Alice performs a complete projective measurement jointly on $A_1A_2A_3A_4$ in the above basis with the measurement outcome ij, then Bob's pair of particles B_1B_2 will be in the state

$$\begin{aligned} &\frac{1}{\sqrt{P_{ij}}}\, {}_{A_1A_2A_3A_4}\left\langle\bar{\prod}^{ij}\right|(|\Psi\rangle_{A_1A_2}\otimes|\chi\rangle_{A_3A_4B_1B_2}) \\ &\quad= \frac{1}{\sqrt{P_{ij}}}\, {}_{A_1A_2A_3A_4}\left\langle\bar{\prod}^{00}\right|[(\sigma^i_{A_1}\otimes\sigma^j_{A_2})|\Psi\rangle_{A_1A_2}\otimes|\chi\rangle_{A_3A_4B_1B_2}] \\ &\quad= \frac{1}{4\sqrt{P_{ij}}}\sum_{i=0,j=0}^{1}\omega^{\kappa}_{ij}|ij\rangle_{B_1B_2}, \end{aligned} \tag{9}$$

where ω^{κ}_{ij} ($\kappa\in\{1,2,3,4\}$) as in Table 1. Here, $|\Psi\rangle_{A_1A_2}\otimes|\chi\rangle_{A_3A_4B_1B_2}$ is the initial complete state of the six particles, A_1, A_2, A_3, A_4, B_1 and B_2. Equation (9) is the analogue of (5). And,

Table 2 Corresponding relations between ij and κ after quantum controlled phase gate operations

ω_{ij}^{κ}	$\kappa=1$	$\kappa=2$	$\kappa=3$	$\kappa=4$
$ij=00$	$a_{00}a$	$\pm^2 a_{01}a$	$\pm^1 a_{10}a$	$\pm^1\pm^2 a_{11}a$
$ij=01$	$\pm^2 a_{01}b$	$a_{00}b$	$\pm^1\pm^2 a_{11}b$	$\pm^1 a_{10}b$
$ij=10$	$\pm^1 a_{10}c$	$\pm^1\pm^2 a_{11}c$	$a_{00}c$	$\pm^2 a_{01}c$
$ij=11$	$\pm^1\pm^2 a_{11}d$	$\pm^1 a_{10}d$	$\pm^2 a_{01}d$	$-a_{00}d$

Table 3 Corresponding relations between ij and κ after the unitary transformation U^{κ}

ω_{ij}^{κ}	$\kappa=1$	$\kappa=2$	$\kappa=3$	$\kappa=4$
$ij=00$	$a_{00}a$	$a_{00}b$	$a_{00}c$	$-a_{00}d$
$ij=01$	$\pm^2 a_{01}b$	$\pm^2 a_{01}a$	$\pm^2 a_{01}d$	$\pm^2 a_{01}c$
$ij=10$	$\pm^1 a_{10}c$	$\pm^1 a_{10}d$	$\pm^1 a_{10}a$	$\pm^1 a_{10}b$
$ij=11$	$\pm^1\pm^2 a_{11}d$	$\pm^1\pm^2 a_{11}c$	$\pm^1\pm^2 a_{11}b$	$\pm^1\pm^2 a_{11}a$

as in [2], the success of this protocol is guaranteed by ${}_{A_1A_2A_3A_4}\langle\overline{\prod}^{00}|\chi\rangle_{A_3A_4B_1B_2}$. Clearly, $P_{ij}=1/16$ and Bob will always succeed in recovering an exact replica of the original state (7) of Alice's particles A_1A_2 after some operations as follows, upon receiving 4 bits of classical information about her measurement result.

(S1) Bob performs a quantum controlled phase gate operation on the particles B_1 and B_2, where the particle B_1 is the control bit and the particle B_2 is the target bit, *i.e.*, if and only if particle B_1 is in the state $|1\rangle$, particle B_2 is performed an operation of Pauli operator (σ_z). Thus Table 1 becomes in Table 2.

(S2) Bob needs to establish a correspondence so that the coefficients a_{ij} correspond to $|ij\rangle$, respectively. Bob performs unitary transformation U^{κ}, and the form of U^{κ} are

$$
\begin{aligned}
U^{\kappa=1} &= \begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix}, \qquad U^{\kappa=2} = \begin{bmatrix} 0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0 \end{bmatrix},\\
U^{\kappa=3} &= \begin{bmatrix} 0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0 \end{bmatrix}, \qquad U^{\kappa=4} = \begin{bmatrix} 0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0 \end{bmatrix}.
\end{aligned} \tag{10}
$$

The unitary transformation U^{κ} will transform Table 2 into Table 3.

(S3) Bob introduces an auxiliary particle A with an initial state $|0\rangle_A$ and makes another unitary transformation U^A on particles (B_1B_2) and A under the basis $\{|000\rangle_{B_1B_2A}$, $|010\rangle_{B_1B_2A}$, $|100\rangle_{B_1B_2A}$, $|110\rangle_{B_1B_2A}$, $|001\rangle_{B_1B_2A}$, $|011\rangle_{B_1B_2A}$, $|101\rangle_{B_1B_2A}$, $|111\rangle_{B_1B_2A}\}$, the unitary transformation U^A may take the form of the following 8×8 matrix:

$$
U^A = \begin{pmatrix} B_1 & B_2 \\ B_2 & -B_1 \end{pmatrix}, \tag{11}
$$

Table 4 Values a_i $(i = 1, 2, 3, 4)$ of the unitary transformation U^A corresponding to the states of particles $B_1 B_2$

States of particles B_1 and B_2	a_1	a_2	a_3	a_4
$\sum_{i=0,j=0}^{1} \omega_{ij}^{1} \vert ij\rangle_{B_1 B_2}$	$\frac{d}{a}$	$\frac{d}{b}$	$\frac{d}{c}$	1
$\sum_{i=0,j=0}^{1} \omega_{ij}^{2} \vert ij\rangle_{B_1 B_2}$	$\pm^2 \frac{d}{b}$	$\pm^2 \frac{d}{a}$	$\pm^2 1$	$\pm^2 \frac{d}{c}$
$\sum_{i=0,j=0}^{1} \omega_{ij}^{3} \vert ij\rangle_{B_1 B_2}$	$\pm^1 \frac{d}{c}$	$\pm^1 1$	$\pm^1 \frac{d}{a}$	$\pm^1 \frac{d}{b}$
$\sum_{i=0,j=0}^{1} \omega_{ij}^{4} \vert ij\rangle_{B_1 B_2}$	$\pm^1 \pm^2 1$	$\pm^1 \pm^2 \frac{d}{c}$	$\pm^1 \pm^2 \frac{d}{b}$	$\pm^1 \pm^2 \frac{d}{a}$

where B_i $(i = 1, 2)$ is a 4×4 matrix and may be written as

$$B_1 = \mathrm{diag}(a_1, a_2, a_3, a_4), \tag{12}$$

$$B_2 = \mathrm{diag}\left(\sqrt{1 - a_1^2}, \sqrt{1 - a_2^2}, \sqrt{1 - a_3^2}, \sqrt{1 - a_4^2}\right), \tag{13}$$

with a_i $(i = 1, 2, 3, 4$ and $|a_i| \le 1)$ depends on the state of particles $(B_1 B_2)$. Table 4 shows all the kinds of different coefficients a_i $(i = 1, 2, 3, 4)$ of the unitary transformation U^A performed by Bob on the states of particles B_1 and B_2. The unitary transformation U^A will transform the state (9) into

$$U_\kappa^A \frac{1}{4\sqrt{P_{ij}}} \sum_{i=0,j=0}^{1} \omega_{ij}^{\kappa} |ij\rangle_{B_1 B_2} \otimes |0\rangle_A. \tag{14}$$

Finally, Bob measures the state of an auxiliary particle A. If the measurement result is $|0\rangle_A$, with the help of Alice's Classical Communication, Bob has successfully realized quantum teleportation with a probability of $|d|^2/16$. Otherwise, the teleportation has failed. We can easily verify that Bob can obtain 16 kinds of states; therefore the total probability of successful teleportation is

$$\left(\frac{d}{4\sqrt{P_{ij}}}\right)^2 = \frac{|d|^2}{16 P_{ij}} = |d|^2. \tag{15}$$

From the above analysis, we can see that, in each case, the total probability of successful teleportation for each receiver is $|d|^2$. If the quantum channel is composed of maximally entangled states, *i.e.*, $|a| = |b| = |c| = |d|$, in this sense, the resulting state is "maximally" different from a pair of Bell states. (In contrast, for a pair of Bell states, there is zero entanglement between $A_3 B_1$ and $A_4 B_2$.) Furthermore, the amount of entanglement between $A_3 B_2$ and $A_4 B_1$ is given by the von Neumann entropy

$$S[\rho_{A_3 B_2}] = -a^2 \log_2 a^2 - b^2 \log_2 b^2 - c^2 \log_2 c^2 + d^2 \log_2 d^2, \tag{16}$$

where $\rho_{A_3 B_2} = \mathrm{tr}_{A_4 B_1}(|\chi\rangle_{A_3 A_4 B_1 B_2}\ {}_{A_3 A_4 B_1 B_2}\langle\chi|)$. Clearly, $S[\rho_{A_3 B_2}]$ has maximum value 1 when $|a| = |b| = |c| = |d| = \frac{1}{2}$. So the total probability of successful teleportation is equals 1.

Like [8], in this paper, $|\chi\rangle_{A_3 A_4 B_1 B_2}$ truly differs from the four-qubit GHZ and W states in that both these states do not enable the teleportation of an arbitrary two-qubit state. Indeed, they are inequivalent under stochastic local operations and classical communication. The sixth-order four-qubit filter $\xi_3^{(4)}$ [17] has nonzero expectation value for $|\chi\rangle_{A_3A_4B_1B_2}$:

$$_{A_3A_4B_1B_2}\langle\chi|\xi_3^{(4)}|\chi\rangle_{A_3A_4B_1B_2} = \frac{1}{2}\sum_{\alpha,\beta,\gamma=0}^{3} E^{\alpha_1\alpha_2}E_{\alpha_1\alpha_2}E^{\beta_1\beta_2}E_{\beta_1\beta_2}E^{\gamma_1\gamma_2}E_{\gamma_1\gamma_2} = 1. \quad (17)$$

Here,

$$E^{\alpha_1\alpha_2} \equiv_{A_3A_4B_1B_2} \langle\chi|\sigma^{\alpha_1}\otimes\sigma^{\alpha_2}\otimes\sigma^2|\chi\rangle_{A_3A_4B_1B_2}, \quad (18)$$

$$E^{\beta_1\beta_2} \equiv_{A_3A_4B_1B_2} \langle\chi|\sigma^{\beta_1}\otimes\sigma^2\otimes\sigma^{\beta_2}\otimes\sigma^2|\chi\rangle_{A_3A_4B_1B_2}, \quad (19)$$

$$E^{\gamma_1\gamma_2} \equiv_{A_3A_4B_1B_2} \langle\chi|\sigma^2\otimes\sigma^{\gamma_1}\otimes\sigma^{\gamma_2}\otimes\sigma^2|\chi\rangle_{A_3A_4B_1B_2}, \quad (20)$$

$$E_{\eta\lambda} = g_{\eta\mu}g_{\lambda\nu}E^{\mu\nu}, \quad (21)$$

$$g_{\mu\nu} \equiv \mathrm{diag}\{-1, 1, 0, 1\}. \quad (22)$$

We note that σ is entangled, whereas any reduced state obtained from a GHZ state is separable. Where the-sixth-order four-qubit filter $\xi_3^{(4)}$ has the expectation values $1/2$ for the GHZ state and 0 for the W state. On the other hand, the third order filter $\xi_1^{(4)}$ and fourth order filter $\xi_2^{(4)}$ have expectation value 1 for GHZ state but yield, respectively, 0 and 1 for $|\chi\rangle_{A_3A_4B_1B_2}$. Note that we are not claiming that $|\chi\rangle_{A_3A_4B_1B_2}$ is LOCC inequivalent to either the GHZ or W state. This would require further work.

In conclusion, we have proposed a protocol for faithful teleportation of an arbitrary, unknown two-particles state from sender to receiver by using $|\chi\rangle_{A_3A_4B_1B_2}$. The results show that for such a $|\chi\rangle_{A_3A_4B_1B_2}$ quantum channel, with the help of Alice's classical communication, there is still a certain probability of successful teleportation. These can similarly be achieved using two Bell pairs. However, by construction, this state is different from a pair of Bell states, because, our four qubit entangled channel can not being reducible to a tensor product of two Bell states. It is a genuine four-partite entangled state, which has properties that differ from those of four-party GHZ and W states. Compared with previous schemes [6–8], the quantum channel is different and the probability of success is determined by the smaller coefficient of the state $|\chi\rangle_{A_3A_4B_1B_2}$ use as the quantum channel. Nowadays, a number of feasible protocols for generating entangled four-particle cluster states [18–20] have been proposed, therefore we believe that this protocol may be realized in the realm of current experimental technology.

Acknowledgement The project supported by National Natural Science Foundation of China under Grant No. 10575017.

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